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COMMUNICATION SYSTEMS ANALYSIS FOR LIGHTWEIGHT ROVING LUNAR VEHICLE

61 SPC-4

SPACE SYSTEMS

DEFENSE SYSTEMS DEPARTMENT • SANTA BARBARA, CALIFORNIA

COMMUNICATION SYSTEMS ANALYSIS FOR LIGHTWEIGHT ROVING LUNAR VEHICLE

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2 November 1961

61 SPC-4

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GENERAL ELECTRIC COMPANY
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INTRODUCTION

This TIS summarizes the results of the analysis carried out in three separate reports contained herein. The three reports are entitled "Communication Requirements" Nos. 1 through 3 respectively and are reproduced in this TIS in their original form.

All three reports analyze transmitter power requirements in terms of data rate for the same system parameters. The system parameters were obtained from a scrutiny of JPL Research Summaries, listed in the references throughout the report.

CONCLUSIONS

Reference to figure 1 of report #1, figure 2 of report #2 and figure 3 of report #3 indicates that transmitter powers of 15, 16, and 5.5 watts respectively are required for the same data rate. These results are tabulated below for ease of comparison. The results are based on the utilization of a 85' receiving antenna.

Table A

Minimum Transmitter Power
Requirements
85' Receiving Antenna

Modulation System	Data Rate	Transmitter Power Watts	dbw
FM	$5x10^5$ elements per/sec($\equiv$ $1.5x10^6$ bits/sec)	5.5	7.4
PCM - PS	$1.5x10^6$ bits/sec	15	11.75
PCM - FM	$1.5x10^6$ bits/sec	16	12

Where the analysis was based on the utilization of a 250' DSIF receiving antenna, the minimum transmitter power requirements for each of the three modulation schemes are tabulated below.

Table B

Minimum Transmitter Power Requirements
250' Receiving Antenna

Modulation Scheme	Data Rate	Transmitter Power Milliwatts	Transmitter Power dbm
FM	5×10^5 elements/sec ($\approx 1.5 \times 10^5$ bits/sec)	370	25.7
PCM - FM	1.5×10^6 bits/sec	940	29.75
PCM - PS	1.5×10^6 bits/sec	1000	30

COMMUNICATION REQUIREMENTS

REPORT # 1

PCM - PS

Preliminary Transmitter R. F. Power Requirements

The purpose of this report is to determine the transmitter power requirements as a function of data transmission rate for the Prospector moon-to-earth TV link.

The analysis that follows is based on the utilisation of PCM-PS modulation at an r. f. carrier frequency of 2250 mc.

The ground based receiver station will utilise the DSIF equipment.

The transmitter power requirements utilising FM and PCM-FM will be made the subject of later reports.

Video Input

The video input will be obtained from the output terminals of an image orthicon camera located aboard the "tank". The maximum data rate assumed is 1.5×10^6 bits/sec, i.e., 1000 vertical lines, 500 horizontal lines and 8 levels of grey,

Video Output

The video output from the communications system will consist of a train of binary pulses that will have a digit error of 1 in 10^5.

System Parameters

Path distance moon-to-earth 2.5×10^5 statute miles

Carrier frequency 2250 mc.

Noise temperature T_g of moon (Ref. 1)	130°K
Diameter of receiving antenna (Ref. 2)	85 ft.

Gain of receiving antenna (Ref. 2), G_r	50 db
Noise temperature, T_r receiving antenna (Ref. 2)	50^oK
Receiving antenna feed and coupling losses L_r (Ref. 2)	0.4 db
Low noise maser amplifier effective tempera ture, T_e (Ref. 3)	30^oK
Transmitting antenna dia. (parabolic)	4 ft.
Transmitting antenna gain, G_t based on 55% efficiency	27 db
Transmitter/antenna coupling and matching losses (assumed), L_t	0.6 db
Circular polarization losses, L_p	3 db

Analysis

The system noise temperature T_s of the receiving system is given by: (Ref. 4)

$$T_s = \left[T_g + T_r\right] + \left[L_r - 1\right] T_o + \left[T_e \;\; L_r\right] \quad (1)$$

where $T_o = 290^o$ K

Hence, we obtain T_s

$$= \left[130 + 50\right] + \left[1.1 - 1\right] \; 290 + \left[30 \times 1.1\right]$$

$$= 242^o \text{ K}$$

The noise power P_n per cycle of post detection bandwidth is

$$P_n = 2KT_s = 1.38 \times 10^{-23} \times 2.42 \times 10^2 \times 2$$

$$= 6.68 \times 10^{-21} \text{ watts/cps}$$

$$= -202 \text{ dbw} \qquad (2)$$

For a digit error of 10^{-5} we require a post detection S/N of 13 db (Ref. 5). The post detection bandwidth here is assumed to be equal to the data rate. If we let this bandwidth be B_n, then the noise power will be $P_{Bn} = 2\ KT_sB_n$ watts (3). Then for a data rate of 1.5×10^6 bits/sec, we obtain from (2) and (3).

$$P_{Bn} = -202 + 10 \log_{10} 1.5 \times 10^6$$
$$= -140.25 \text{ dbw}$$

Since there is no modulation gain in a PCM-PS system, then the received signal input power P_r, required for a 13 db output S/N is

$$P_r = P_{Bn} + 13 \text{ db} \quad (4)$$
$$= -140.25 + 13 = -127.25 \text{ dbw}$$

The received signal input power P_r is given by

$$P_r = P_t - L_t + G_t - \alpha + G_r - L_r - L_p \quad (5)$$

Where P_t is transmitter power level (ref to 1 watt)

α is the free space loss between isotropic antennas and is given by

$$\alpha = 37 + 20 \log f\,(\text{mc}) + 20 \log d\,(\text{miles})$$

for $f = 2250$ mc and $d = 2.5 \times 10^5$ miles

we obtain $\alpha = 212$ db

The transmitter power required P_t is from (5)

$$P_t = P_r + L_t + \alpha + L_r + L_p - G_t - G_r$$

and for the 1.5×10^6 bit/sec data we get

$$P_t = -127.25 + 0.6 + 212 + 0.4 + 3 - 27 - 50$$
$$= 11.75 \text{ dbw} = 15 \text{ watts}$$

Values of P_t required for data rates between 5×10^4 and 1.5×10^6 bits/sec are tabulated in Table 1. The results of Table 1 are plotted in Figure 1.

Case for 250 Foot Antenna at DSIF

If the 250'dish is utilized at the DSIF site, the system parameters are changed as follows: (Ref. 2)

Receiving antenna gain G_r increased to 61 db.

Receiving antenna noise temperature T_a, reduced to 15° K.

The system noise temperature T_s, is reduced by 35° K to 242-35 = 207° K and the thermal noise power per cycle of bandwidth is reduced from -202 dbw to $-202 - 10 \log \frac{242}{207} = 202.7$ dbw.

The noise N_{Bn} in a 1.5 mc bandwidth becomes -(140.25 + 0.7) = say -141 dbw and the received signal power required, P_r = -141 + 13 = -128 dbw.

The system transmission gain, however, is increased by 11 db due to the higher receiving antenna gain. The net result is that the transmitter power requirements are reduced by 11 + 0.7 = 11.7 db.

Transmitter power requirements as a function of data rate utilizing the 250 ft. DSIF antenna, are tabulated in Table II.

The results of Table II are plotted in Figure 2.

References

1. G. E. Co., DED presentation "Techniques Applicable to Lunar Landing", Feb. 10, 1960. Page 27, Chart 6-1

2. JPL Technical Memo 33-27, Feb. 13, 1961. Page 22

3. JPL Research Summary 36-7, Vol. 1, Feb. 15, 1961. Page 78

4. H. I. Ewen, "A Thermodynamic Analysis of Maser Systems", Microwave J, Vol. 2, Pages 41-46, March 1959.

5. H. N. Putschi, "Evaluation and Development of a PCM-PS Radio Telemetry System". G. E. Co., TIS R59ELS34, May 7, 1959. Figure 16

6. Reference Data for Radio Engineers, I.T.T.L, 4th edition. Page 751

Table I

Transmitter Power Vs. Data Rate, 85 ft. Antenna

Data Rate, bits/sec	Noise Band-width, B_n	Noise Power P_{Bn} in B_n (dbw)	Output S/N for $P_e = 10^{-5}$ (db)	Rec'd. signal power P_r(dbw)	Net Trans-mission loss N_L (db)	Trans-mitter power P_t(dbw)	P_t Watts
1.5×10^6	1.5 mc	-140.25	13	-127.25	-139	11.75	15
1.0×10^6	1 mc	-142	13	-129	-139	10	10
5×10^5	500 kc	-145	13	-132	-139	7	5
2.5×10^5	250 kc	-148	13	-135	-139	4	2.5
1×10^5	100 kc	-152	13	-139	-139	0	1.0
5×10^4	50 kc	-155	13	-142	-139	-3	0.5

Table II

Transmitter Power Vs. Data Rate, 250 ft. Antenna

Data Rate bits / sec	Noise Band-width, B_n	Noise Power P_{Bn} (dbw)	O/P S/N for $P_e = 10^{-5}$ (db)	Rec'd. signal power P_r (dbw)	Net trans-mission loss (db)	Transmitter Power		
						dbw	dbm	milli-watts
1.5×10^6	1.5 mc	-141	13	-128	-128	0	30	1000
1.0×10^6	1 mc	-142.7	13	-129.7	-128	-1.7	28.3	680
5×10^5	500 kc	-145.7	13	-132.7	-128	-4.7	25.3	340
2.5×10^5	250 kc	-148.7	13	-135.7	-128	-7.7	22.3	170
1×10^5	100 kc	-152.7	13	-139.7	-128	-11.7	18.3	68
5×10^4	50 kc	-155.7	13	-142.7	-128	-14.7	15.3	34

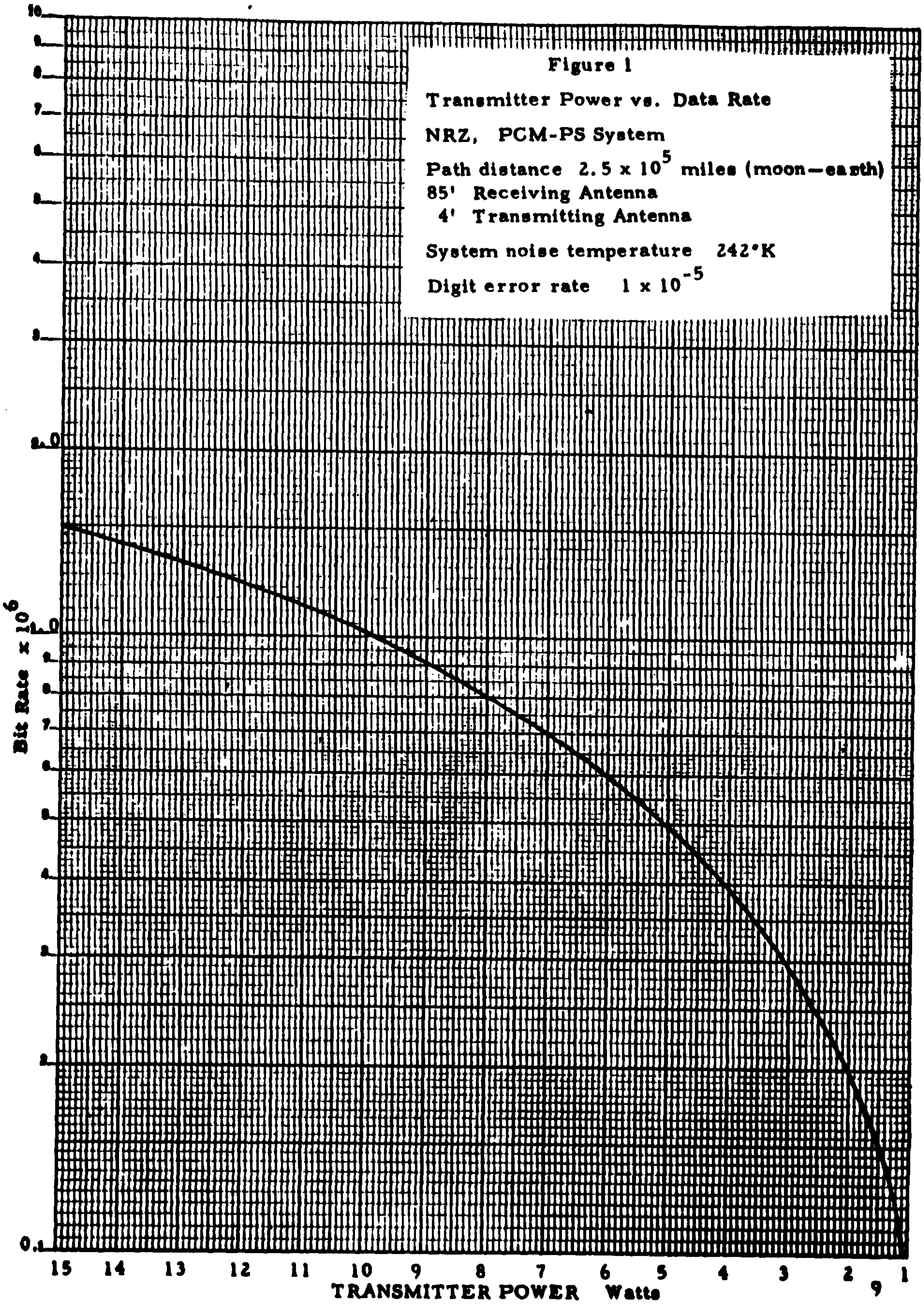


Figure 1
Transmitter Power vs. Data Rate
NRZ, PCM-PS System
Path distance 2.5 x 10^5 miles (moon—earth)
85' Receiving Antenna
4' Transmitting Antenna
System noise temperature 242°K
Digit error rate 1 x 10^-5
Bit Rate x 10^6
10
2.0
1.0
0.1
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
TRANSMITTER POWER Watts

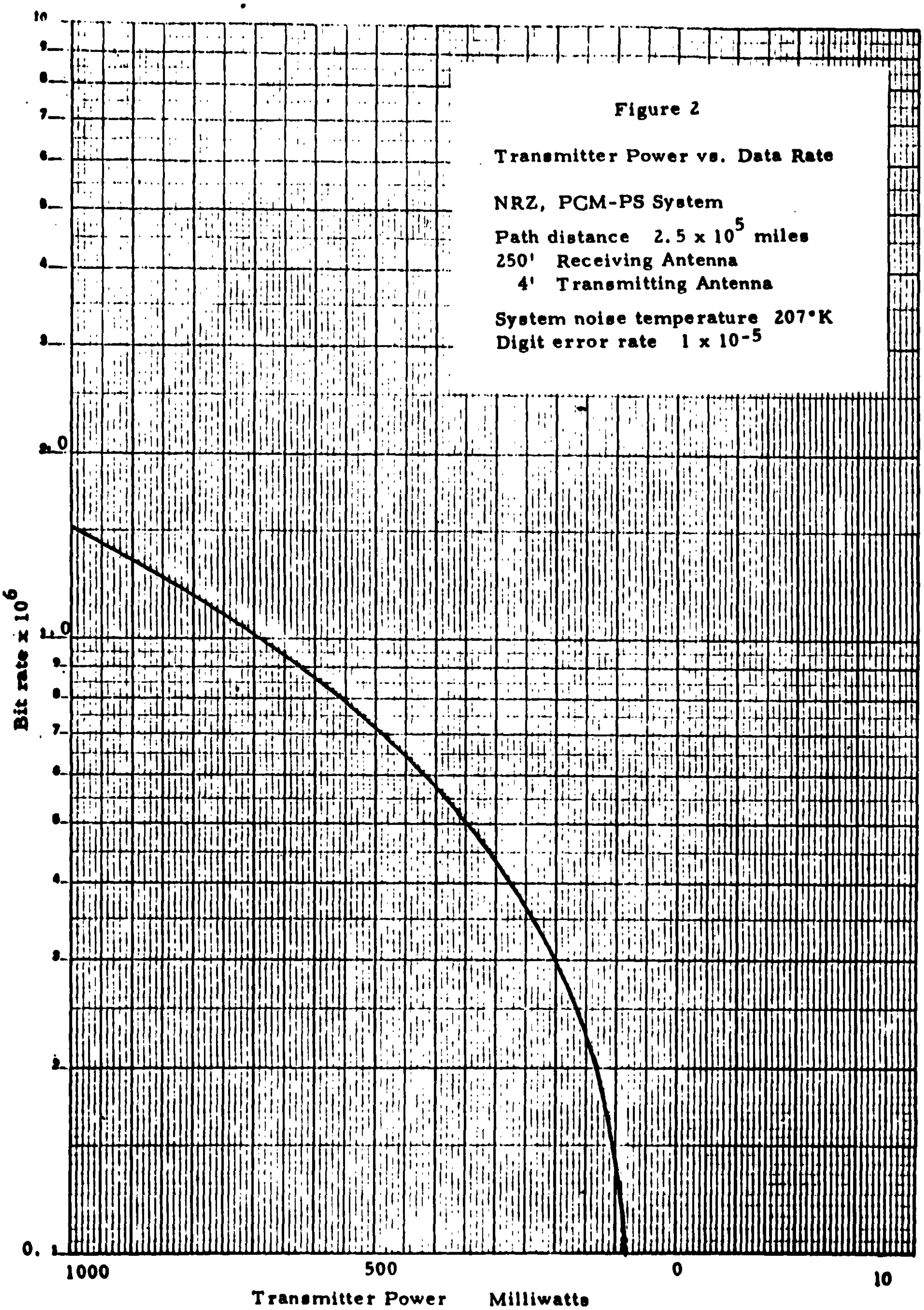


Figure 2

Transmitter Power vs. Data Rate

NRZ, PCM-PS System

Path distance 2.5×10^5 miles
250' Receiving Antenna
4' Transmitting Antenna

System noise temperature 207°K
Digit error rate 1×10^{-5}

COMMUNICATION REQUIREMENTS

REPORT #2
PCM - FM

Preliminary Transmitter RF Power Requirements

This report discusses the transmitter power requirements as a function of data rate for the Prospector moon-earth T. V. link.

The analysis that follows is based on the utilization of a PCM-FM modulation system at a carrier frequency of 2250 mc.

The ground based receiver station will utilize DSIF equipment.

System Parameters

Path distance moon-earth 2.5×10^5 statute miles

Carrier frequency - 2250 mc

Noise temperature T_g of moon [1]	130° K
Gain of 85' receiving antenna [2] G_r	50 db
Noise temperature, T_r of 85' receiving antenna [2]	50° K
Gain of 250' receiving antenna, [2] G_r	61 db
Noise temperature T_r of 250' receiving antenna [2]	15° K
Receiving antenna feed and coupling losses, L_r [2]	0.4 db
Circular polarization losses, L_p	3 db
Gain of 4' diameter transmitting antenna, G_r	27 db

Transmitting antenna coupling and other losses (assumed) 0.6 db

Maser amplifier effective temperature,[3] T_e 30° K

Analysis

The system noise temperature T_s is given by[4]

$$T_s = \left[T_g + T_r\right] + \left[L_r - 1\right] T_o + \left[T_e L_r\right] \tag{1}$$

where $T_o = 290°\ K$

and for the 85' antenna system we obtain

$$T_s = 242°\ K$$

For the 250' antenna system we get $T_s = 207°\ K$.

The thermal noise P_n in the I. F. bandwidth, B_{IF} is

$$P_n = KT_s B_{IF} \tag{2}$$

$$= 1.38 \times 10^{-23}\ T_s B_{IF}$$

for the 85' antenna system we get

$$P_n = 3.34 \times 10^{-21} \text{ watts/cycle} = \underline{\underline{-205}} \text{ dbw/cps}$$

and for the 250' antenna system we get

$$P_n = \underline{\underline{-205.7}} \text{ dbw/cps}$$

The post detection output S/N of a FM system is given by (See Appendix I)

$$S/N_{db} = 10 \log_{10} \frac{P_R}{P_n} + 10 \log_{10} \left(\frac{B_{IF}}{2\,Bv}\right) + 20 \log_{10} \left(\frac{\Delta F}{f_m}\right) + 5 - 3 \tag{3}$$

Where P_r is the received signal power

P_n is the thermal noise power in the I. F. bandwidth

B_{IF} is the I. F. bandwidth

B_v is the post detection bandwidth

ΔF is the peak deviation of the r. f. carrier

f_m is the highest modulating frequency (= B_v)

The +5 db term is the triangular noise spectrum improvement factor, characteristic of a F. M. discriminator.

The -3 db term is the efficiency correction factor for imperfect limiting.

The ratio ($\frac{\Delta F}{f\,m}$) is the modulation index of the system.

F. M. Improvement Threshold

Equation (3) is only valid provided that the signal level in the I. F. amplifier is above the improvement threshold of the receiver. T e improvement threshold is defined as the signal level where the peak signal is equal to or greater than the peak noise. Since the input noise is assumed to be "white noise" it has a Gaussian distribution. Reference to Figure 1 shows that for 99. 9% of the time the peak to RMS voltage factor does not exceed approximately 10 db. We therefore define the improvement threshold as the "10 db threshold". The received signal power required

$$P_R \therefore = P_n + 10 \text{ db} \tag{4}$$

I. F. Bandwidth

The I. F. bandwidth, B_{IF}, is given by [5]

$$B_{IF} = 2 \ (\Delta F + B_v) \tag{5}$$

Equation (5) states that the I. F. bandwidth required is twice the sum of the peak deviation and the video bandwidth. It should be noted that doppler shift and frequency drift is neglected.

Video Bandwidth

In order to make a fair comparison between the PCM-PS system discussed in Report #1 and a PCM-FM system we will assume that the video bandwidth B_v for the FM case is the same as for the PS case, i.e., B_v equal to the data rate.

Transmitter power V_s data rate is tabulated in Tables I and II for modulation indices of 1 and 2.4 respectively utilizing 85' receiving antennas. Tables III and IV indicate transmitter power requirements utilizing 250' receiving antennas.

The results of Tables I through IV are plotted in Figures 2 - 5 respectively.

References

(1) G. E., D. E. D. Presentation "Techniques Applicable to a Lunar Landing", February 19, 1960, page 27, Chart 6-1.

(2) J. P. L. Technical Memorandum 33-27, February 13, 1961, page 22.

(3) J. P. L. Research Summary 36-7, Volume I, February 15, 1961, page 78.

(4) H. I. Ewen, "A Thermodynamic Analysis of Maser Systems" Microwave J., Volume 3, March 1959, pages 41 - 46.

(5) H. S. Black, "Modulation Theory" D. Van Nostrand Co., November 1958, pages 200 - 202.

TABLE I

TRANSMITTER POWER VS DATA RATE

PCM-FM 85' RECEIVING ANTENNA

Modulation Index of 1

Data Rate bits/sec	Video Band-width	Peak De-viation ΔF	IF Band-width (= $2(\Delta F + B_v)$) B_{if}	Noise Power in B_{if}, P_n I. F. (dbw)	Received Signal Power P_R (dbw)	Net Trans-mission Loss (db)	Transmitter Power dbw	Transmitter Power Watts	O/P S/N (db)
1.5×10^6	1.5 mc	1.5 mc	6 mc	- 137	- 127	139	12	16	15
1.0×10^6	1 mc	1 mc	4 mc	- 139	- 129	139	10	10	15
5×10^5	500 kc	500 kc	2 mc	- 142	- 132	139	7	5	15
2.5×10^5	250 kc	250 kc	1 mc	- 145	- 135	139	4	2.5	15
1×10^5	100 kc	100 kc	400 kc	- 149	- 139	139	0	1	15
5×10^4	50 kc	50 kc	200 kc	- 152	- 142	139	-3	0.5	15

Probability of error, P_e, $< 10^{-7}$

TABLE II

TRANSMITTER POWER VS DATA RATE

PCM-FM 85' RECEIVING ANTENNA

Modulation Index of 2.4

Data Rate bits/sec	Video Band-width B_v	ΔF Peak Deviation	I. F. Band-width B_{if}	Noise Power in B_{if} (dbw)	Received Signal Power (dbw)	Net Trans-mission Loss (db)	Transmitter Power dbw	Transmitter Power Watts	Output S/N (db)
1.5×10^6	1.5 mc	3.6 mc	10.2 mc	-135	-125	139	14	25	25
1.0×10^6	1 mc	2.4 mc	6.8 mc	-137	-127	139	12	16	25
5×10^5	500 kc	1.2 mc	3.4 mc	-140	-130	139	9	8	25
2.5×10^5	250 kc	600 kc	1.7 mc	-143	-133	139	6	4	25
1×10^5	100 kc	240 kc	680 kc	-147	-137	139	2	1.6	25
5×10^4	50 kc	120 kc	340 kc	-150	-140	139	-1	0.8	25

$P_e \ll 10^{-7}$

TABLE III

TRANSMITTER POWER VS DATA RATE

PCM-FM, 250' RECEIVING ANTENNA

Modulation Index of 1

Data Rate bits/sec	Video Band-width B_v	Peak Deviation ΔF	I. F. Band-width B_{if}	Noise Power in B_{if} (dbw)	Received Signal Power P_R (dbw)	Net Trans-mission Loss (db)	Transmitter Power (dbw)	Transmitter Power (dbm)	Transmitter Power (milli-watts)	Output S/N (db)
1.5×10^6	1.5 mc	1.5 mc	6 mc	-137.73	-127.75	128	0.25	29.75	940	15
1.0×10^6	1 mc	1 mc	4 mc	-139.75	-129.75	128	-1.75	28.25	670	15
5×10^5	500 kc	500 kc	2 mc	-142.75	-132.75	128	-4.75	25.25	335	15
2.5×10^5	250 kc	250 kc	1 mc	-145.75	-135.75	128	-7.75	22.25	168	15
1.0×10^5	100 kc	100 kc	400 kc	-149.75	-139.75	128	-11.75	18.25	67	15
5×10^4	50 kc	50 kc	200 kc	-152.75	-142.75	128	-14.75	15.25	33.5	15

$P_e < 10^{-7}$

TABLE IV

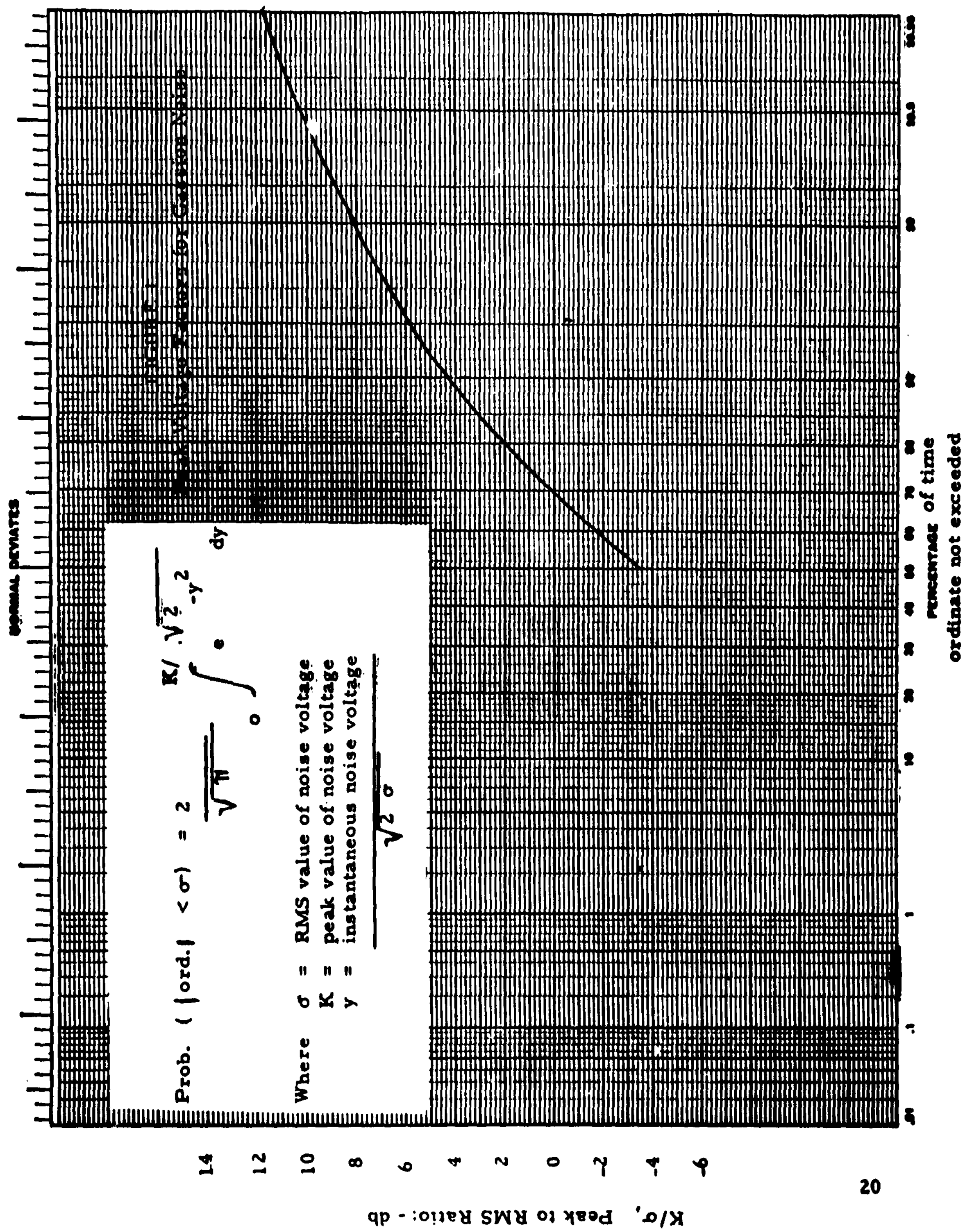
TRANSMITTER POWER VS DATA RATE

PCM-FM, 250' RECEIVING ANTENNA

Modulation Index of 2.4

Data Rate bits/sec	Video Band-width B_v	Peak Deviation ΔF	I. F. Band-width B_{if}	Noise Power in B_{if} (dbw)	Received Signal Power P_R (dbw)	Net Trans-mission Loss (db)	Transmitter Power (dbw)	Transmitter Power (dbm)	Transmitter Power (milli-watts)	Output S/N (db)
1.5×10^6	1.5 mc	3.6 mc	10.2 mc	-135.75	-125.75	128	2.25	32.25	1680	25
1×10^6	1 mc	2.4 mc	6.8 mc	-137.75	-127.75	128	0.25	30.25	1060	25
5×10^5	500 kc	1.2 mc	3.4 mc	-140.75	-130.75	128	-2.75	27.25	530	25
2.5×10^5	250 kc	600 kc	1.7 mc	-143.75	-133.75	128	-5.75	24.25	265	25
1×10^5	100 kc	240 kc	680 kc	-147.75	-137.75	128	-9.75	20.25	106	25
5×10^4	50 kc	120 kc	340 kc	-150.75	-140.75	128	-12.75	17.25	53	25

$P_e << 10^{-7}$


NORMAL DEVIATES
Prob. (| ord. | < σ) = 2/√π ∫₀^{K/√2σ} e^{-y²} dy
Where σ = RMS value of noise voltage
K = peak value of noise voltage
y = instantaneous noise voltage / √2 σ
14
12
10
8
6
4
2
0
-2
-4
-6
K/σ, Peak to RMS Ratio: - db
PERCENTAGE of time
ordinate not exceeded

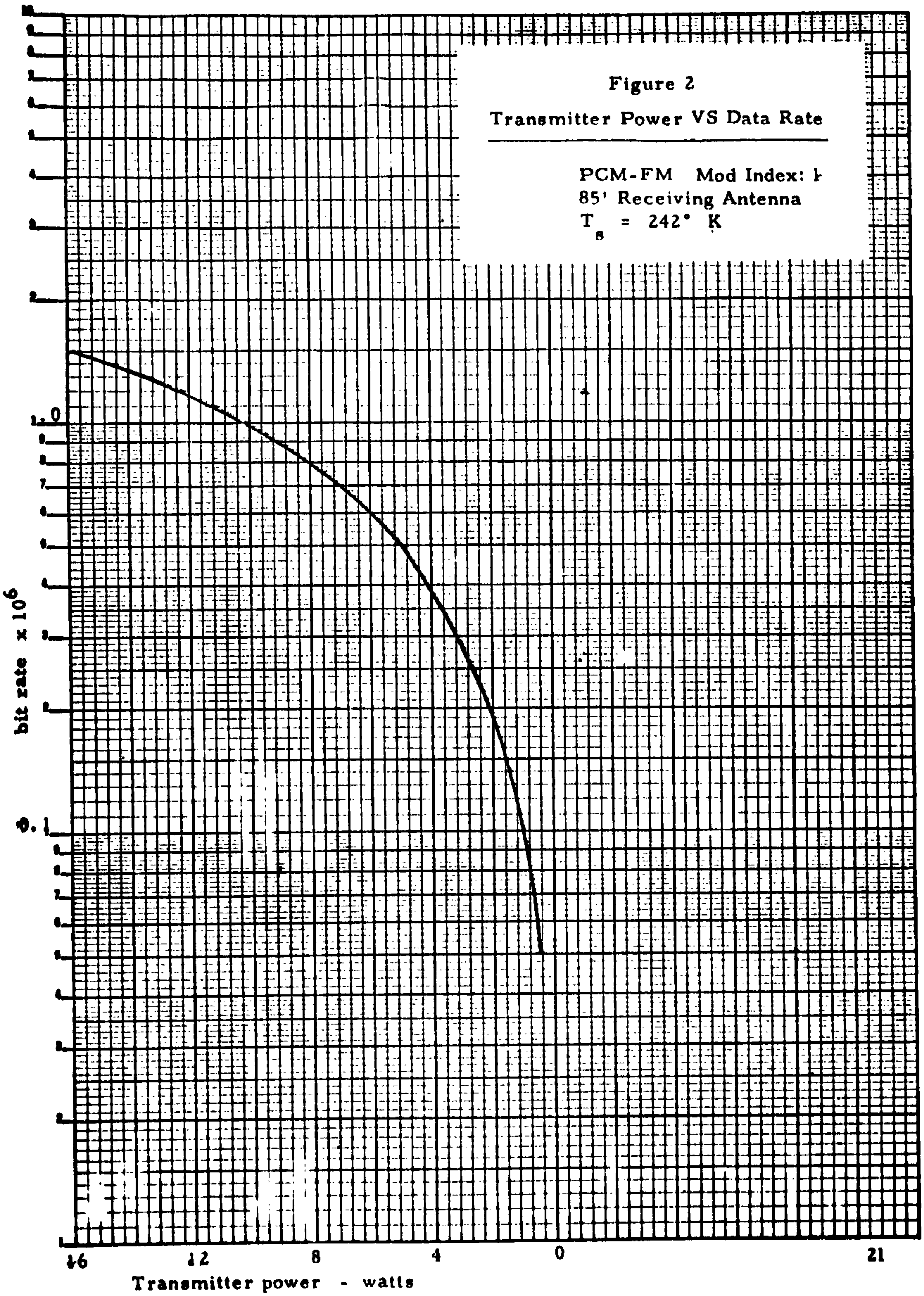


Figure 2
Transmitter Power VS Data Rate
PCM-FM Mod Index: 1
85' Receiving Antenna
T_s = 242° K
bit rate x 10^6
1.0
0.1
16
12
8
4
0
21
Transmitter power - watts

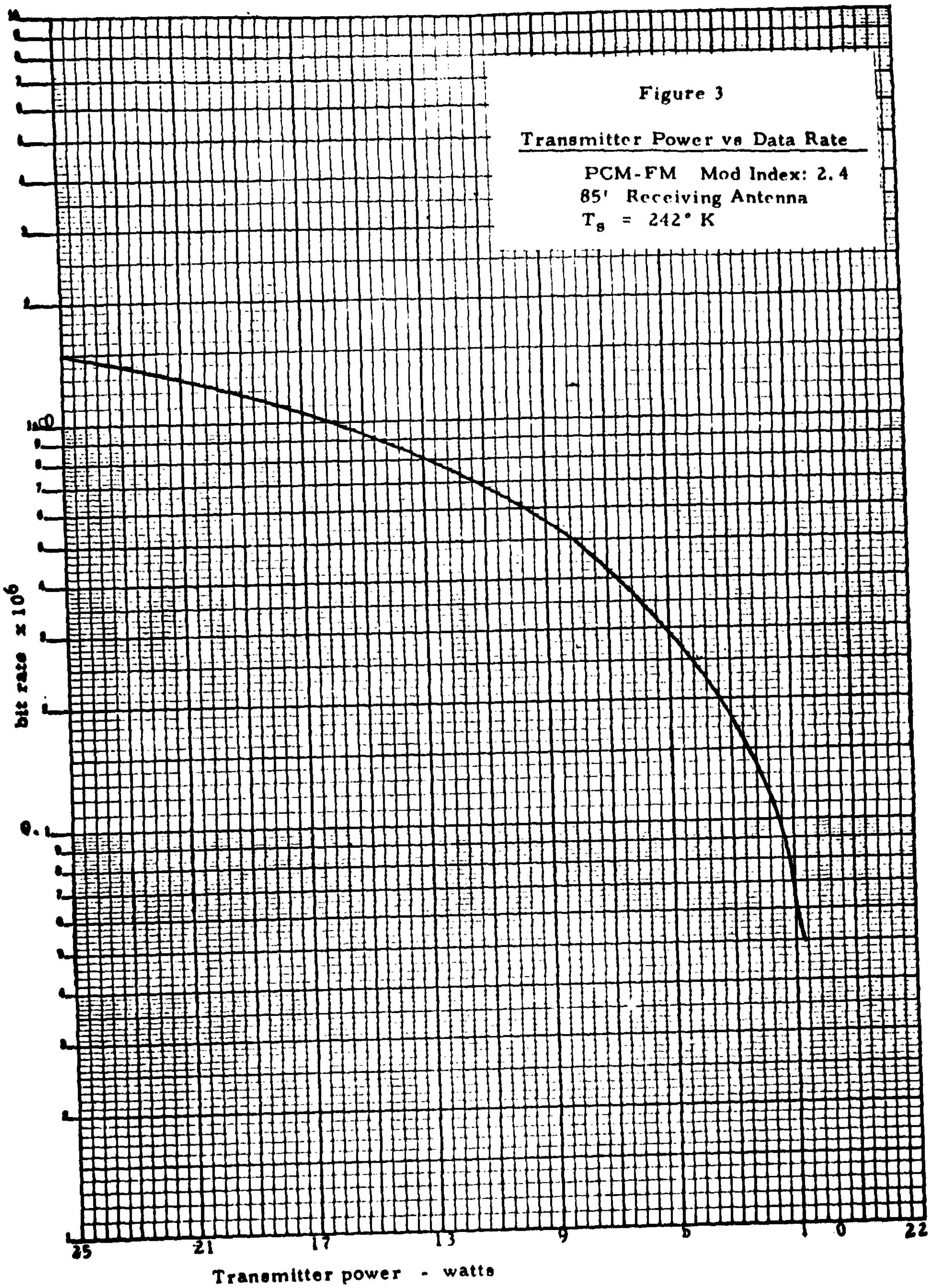


Figure 3

Transmitter Power vs Data Rate

PCM-FM Mod Index: 2.4
85' Receiving Antenna
T_s = 242° K

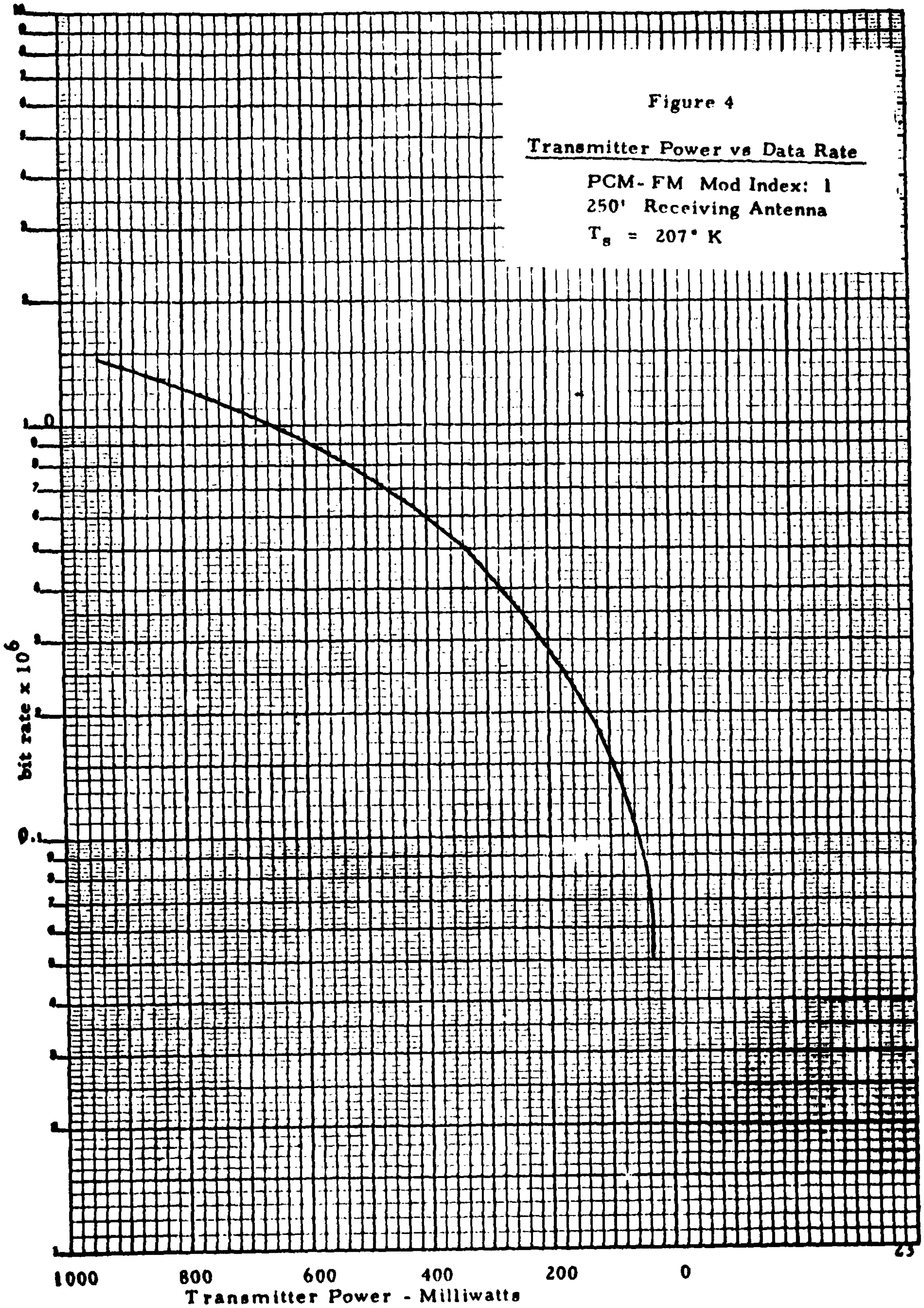


Figure 4
Transmitter Power vs Data Rate
PCM-FM Mod Index: 1
250' Receiving Antenna
T_s = 207° K
bit rate x 10^6
1.0
0.1
1000
800
600
400
200
0
Transmitter Power - Milliwatts

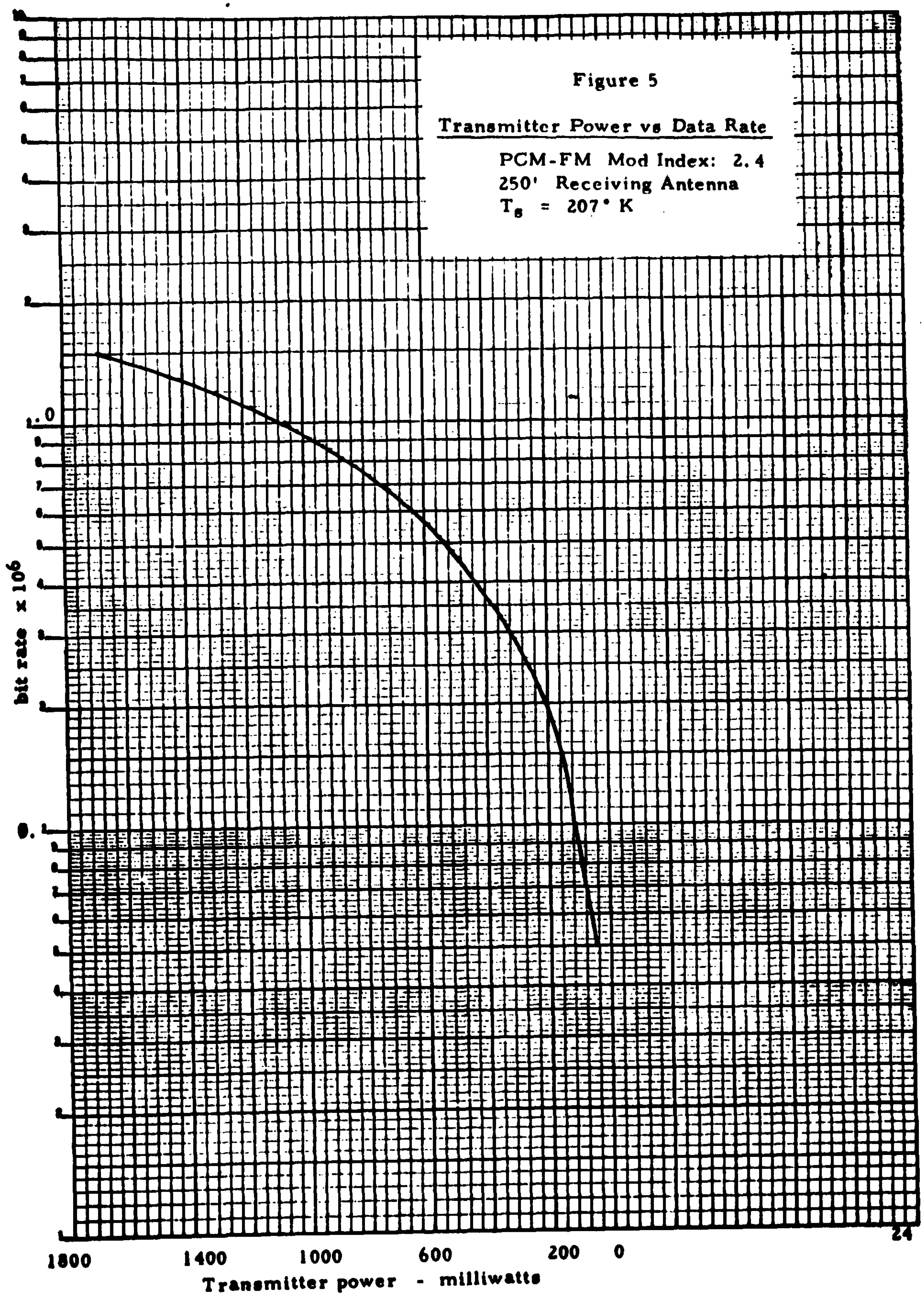


Figure 5
Transmitter Power vs Data Rate
PCM-FM Mod Index: 2.4
250' Receiving Antenna
T_s = 207° K
1.0
0.1
bit rate x 10^6
1800
1400
1000
600
200
0
Transmitter power - milliwatts

COMMUNICATION REQUIREMENTS

REPORT No. 3

F. M.

Preliminary Transmitter RF Power Requirements

The purpose of this report is to determine the transmitter power requirements as a function of signalling rate for the Prospector moon-to-earth TV link.

The analysis that follows is based on a composite video signal frequency modulating (FM) an RF carrier.

The carrier frequency assumed is 2250 Mc and the ground-based receiver station will utilize the DSIF equipment.

Video Input

The video input signal to the communication transmitter will be derived from the output of the TV camera. The input signal will be a composite video containing both the video and synchronizer signals. The video signal will be in analog form and the synch signals will be in the form of pulses.

Video Output

The video output from the ground-based communication receiver will be a single-ended composite video signal.

System Parameters

Path distance moon-to-earth	2.5×10^5 statute miles
Carrier frequency	2250 Mc

Noise temperature T_g of moon[1]	130°K
Diameter of receiving antenna [2]	85 ft
Gain of receiving antenna[2], G_r	50 db
Noise temperature, T_r, [2] receiving antenna	50° K
Receiving antenna feed and coupling losses L_r	0.4 db
Low noise maser amplifier effective temperature, T_e [3]	30° K
Transmitting antenna dia. (parabolic)	4 ft
Transmitting antenna gain, G_t, based on 55% efficiency	27 db
Transmitter/antenna coupling and matching losses (assumed), L_t	0.6 db
Circular polarization losses, L_p	3 db

Output S/N

In order to make a fair comparison between FM, PCM-PS and PCM-FM, equal video output S/N ratios will be assumed. In the PCM-PS and PCM-FM cases (see reports 1 and 2) 8 levels of gray were assumed requiring therefore 3 binary bits. The peak-to-peak signal to rms noise voltage therefore was[4]

$$S/N = 2\sqrt{3}\, S \tag{1}$$

where S = the number of quantum steps

so we obtaine

$$S/N = 2\sqrt{3}\; 8 = 27.7 \quad \text{say } 30{:}1 = 30 \text{ db.}$$

For the purpose of analysis in the FM case, let us assume a composite block negative video signal with a peak-to-peak white voltage $V_{pp} = 1$ and black level $= 0.25 V_{pp}$.

The output S/N therefore taking into account the synch pulses

$$= \frac{30}{.75} = 40:1 = 32 \text{ db.}$$

Analysis

The output S/N $\left(\frac{\text{rms signal}}{\text{rms noise}}\right)$ in an FM system is given by [5]

$$S/N = 10 \log_{10} \frac{P_R}{P_N} + 20 \log \left(\frac{\Delta F}{B_V}\right) + 10 \log \left(\frac{B_{1F}}{2B_V}\right) + 2 \text{ db} \quad (2)$$

The peak-to-peak signal/rms noise ratio is therefore

$$S_{PP}/N = 10 \log \frac{P_R}{P_N} + 20 \log \left(\frac{\Delta F^1}{B_V}\right) + 10 \log \left(\frac{B_{1F}}{2B_V}\right) + 2 \quad (3)$$

where
- P_R = the received signal power
- P_N = the noise power in the I F amplifier
- $\Delta F^1 = 2\Delta F$ = twice the peak deviation of the carrier
- B_{1F} = the I F bandwidth
- B_V = the video bandwidth

2 db represents the difference between the 5 db triangular noise spectrum gain and - 3 db limiter efficiency factor.

The results of the analysis are tabulated on Tables I through IV and plotted in Figures 1 through 4.

TABLE I

TRANSMITTER POWER VS SIGNALLING RATE

FM, MOD INDEX : 1
85' RECEIVING ANTENNA
T_s = 242°K

Signalling Rate (Elements/Sec)	Video Band-width B_v	Peak Deviation ΔF	I F Band-width B_{IF}	Noise Power P_N, in B_{IF}	Recvd Signal Power P_R (dbw)	Net Trans-mission Loss (db)	Transmitter Power (dbw)	(watts)	Output* S/N (db)
$5x10^5$	250 Kc	250 Kc	1 Mc	-145	-124	139	15	31.6	32
$2.5x10^5$	125 Kc	125 Kc	500 Kc	-148	-127	139	12	15.8	32
$1x10^5$	50 Kc	50 Kc	200 Kc	-152	-131	139	8	6.3	32
$5x10^4$	25 Kc	25 Kc	100 Kc	-155	-134	139	5	3.15	32
$1x10^4$	5 Kc	5 Kc	20 Kc	-162	-141	139	-2	0.63	32

*Peak-to-peak signal/rms noise

T_s = System Noise Temperature

TABLE II

TRANSMITTER POWER VS SIGNALLING RATE

FM, MOD INDEX : 1
250' Receiving Antenna

T_s = 207°K

Signalling Rate (Elements/Sec)	Video Bandwidth B_v	Peak Deviation ΔF	IF Band-width B_{IF}	Noise Power in B_{IF} P_N (dbw)	Recvd Signal Power P_R (dbw)	Net Trans-mission Loss (db)	Transmitter Power (dbw)	Transmitter Power (dbm)	Transmitter Power (mw)	Output* S/N (db)
$5x10^5$	250 Kc	250 Kc	1 Mc	-145.7	-124.7	128	3.3	33.3	2140	32
$2.5x10^5$	125 Kc	125 Kc	500 Kc	-148.7	-127.7	128	0.3	30.3	1070	32
$1x10^5$	50 Kc	50 Kc	200 Kc	-152.7	-131.7	128	-3.7	26.3	428	32
$5x10^4$	25 Kc	25 Kc	100 Kc	-155.7	-134.7	128	-6.7	23.3	214	32
$1x10^4$	5 Kc	5 Kc	20 Kc	-162.7	-141.7	128	-13.7	16.3	43	32

* Peak-to-peak signal/ rms noise

T_s = System Noise Temperature

TABLE III

TRANSMITTER POWER VS SIGNALLING RATE

FM, MOD INDEX : 2.4
85' Receiving Antenna
T_s = 242°K

Signalling Rate (Elements/Sec)	Video Bandwidth B_v	Peak Deviation ΔF	IF Bandwidth B_{IF}	Noise Power in B_{IF} P_N (dbw)	Recvd Signal Power P_R (dbw)	Net Transmission Loss (db)	Transmitter Power (dbw)	Transmitter Power (watts)	Output* S/N (db)
5×10^5	250 Kc	600 Kc	1.7 Mc	-142.7	-131.6	139	7.4	5.5	32
2.5×10^5	125 Kc	300 Kc	850 Kc	-145.7	-134.6	139	4.4	2.75	32
1×10^5	50 Kc	120 Kc	340 Kc	-149.7	-138.6	139	0.4	1.1	32
5×10^4	25 Kc	60 Kc	170 Kc	-152.7	-141.6	139	-2.6	.55	32
1×10^4	5 Kc	12 Kc	34 Kc	-159.7	-148.6	139	-9.6	0.11	32

*Peak-to-peak Signal/rms Noise
T_s = System Noise Temperature

TABLE IV

TRANSMITTER POWER VS SIGNALLING RATE

FM, MOD INDEX : 2.4
250' Receiving Antenna

T_s = 207°K

Signalling Rate (Elements/Sec)	Video Band-width B_v	Peak Deviation ΔF	I F Band-width B_{IF}	Noise Power in B_{IF} P_N (dbw)	Recvd Signal Power P_R (dbw)	Net Trans-mission Loss (db)	Transmitter Power (dbw)	Transmitter Power (dbm)	Transmitter Power (mw)	Output* S/N (db)
5×10^5	250 Kc	600 Kc	1.7 Mc	-143.4	-132.3	128	- 4.3	25.7	370	32
2.5×10^5	125 Kc	300 Kc	850 Kc	-146.4	-135.3	128	- 7.3	22.7	185	32
1×10^5	50 Kc	120 Kc	340 Kc	-150.4	-139.3	128	-11.3	18.7	74	32
5×10^4	25 Kc	60 Kc	170 Kc	-153.4	-142.3	128	-14.3	15.7	37	32
1×10^4	5 Kc	12 Kc	34 Kc	-160.4	-149.3	128	-21.3	8.7	7.4	32

*Peak-to-peak Signal/rms noise

T_s = System Noise Temperature

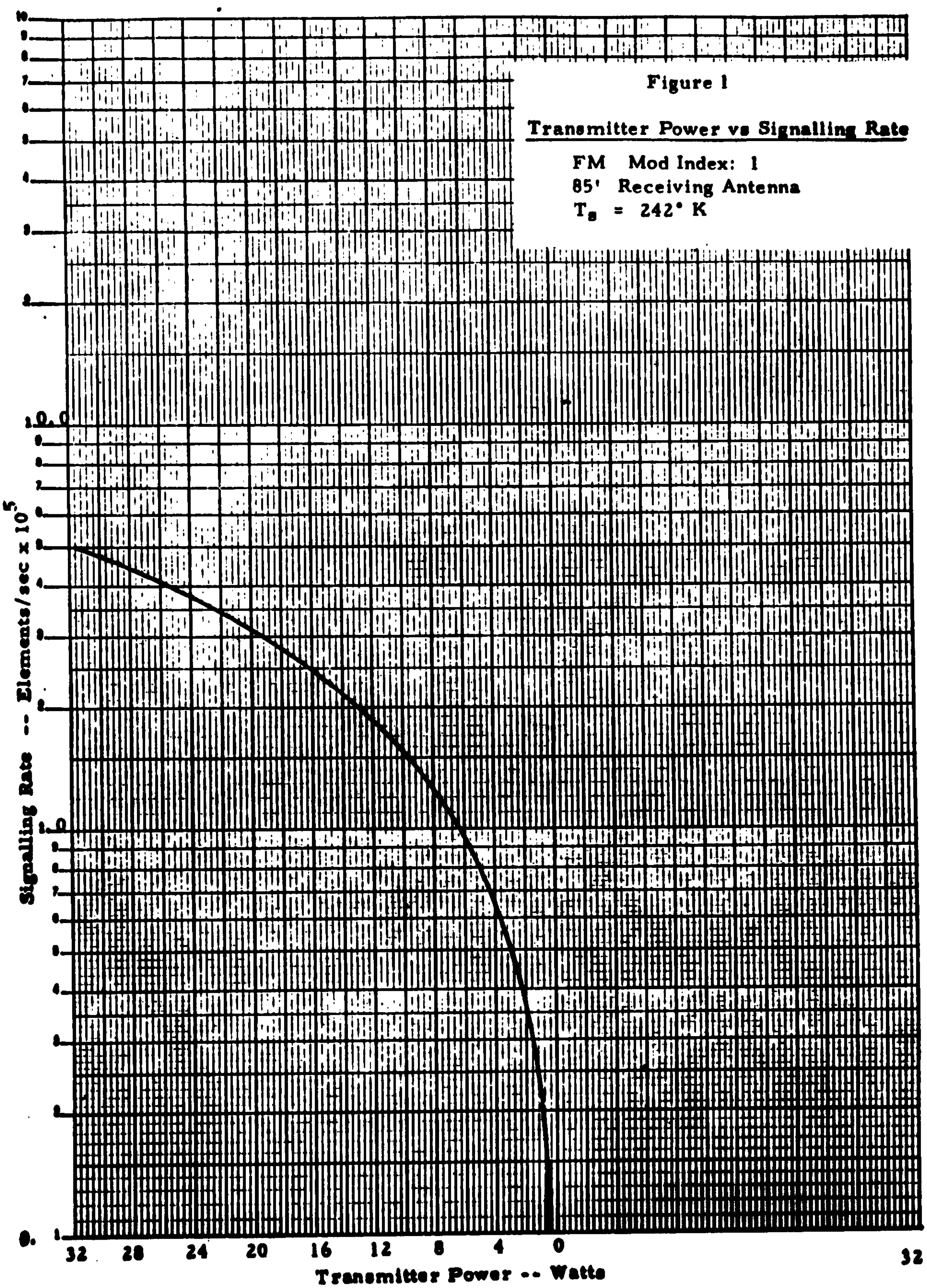


Figure 1
Transmitter Power vs Signalling Rate
FM Mod Index: 1
85' Receiving Antenna
T_s = 242° K
Signalling Rate -- Elements/sec x 10^5
10.0
1.0
0.1
32
28
24
20
16
12
8
4
0
32
Transmitter Power -- Watts

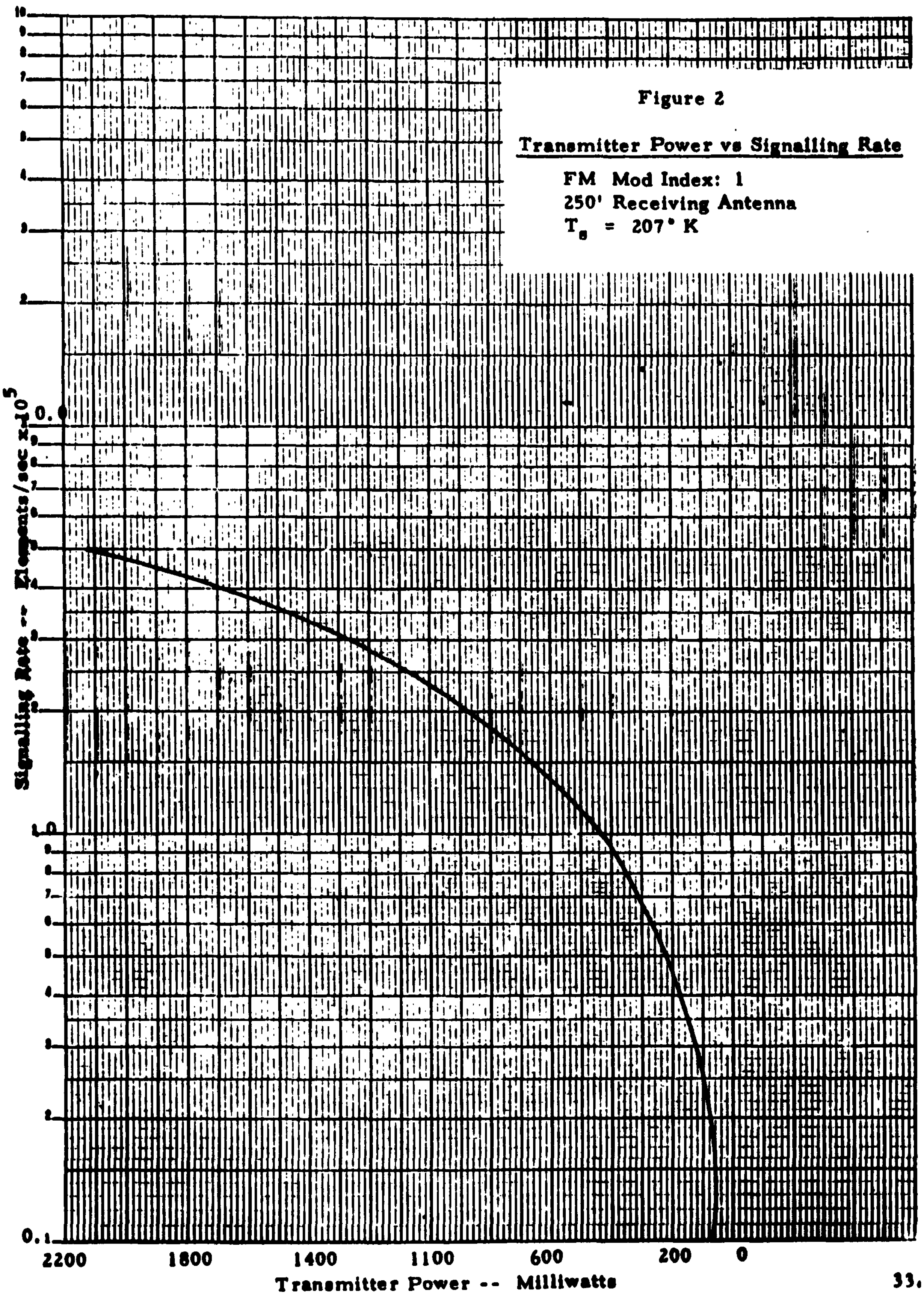


Figure 2

Transmitter Power vs Signalling Rate

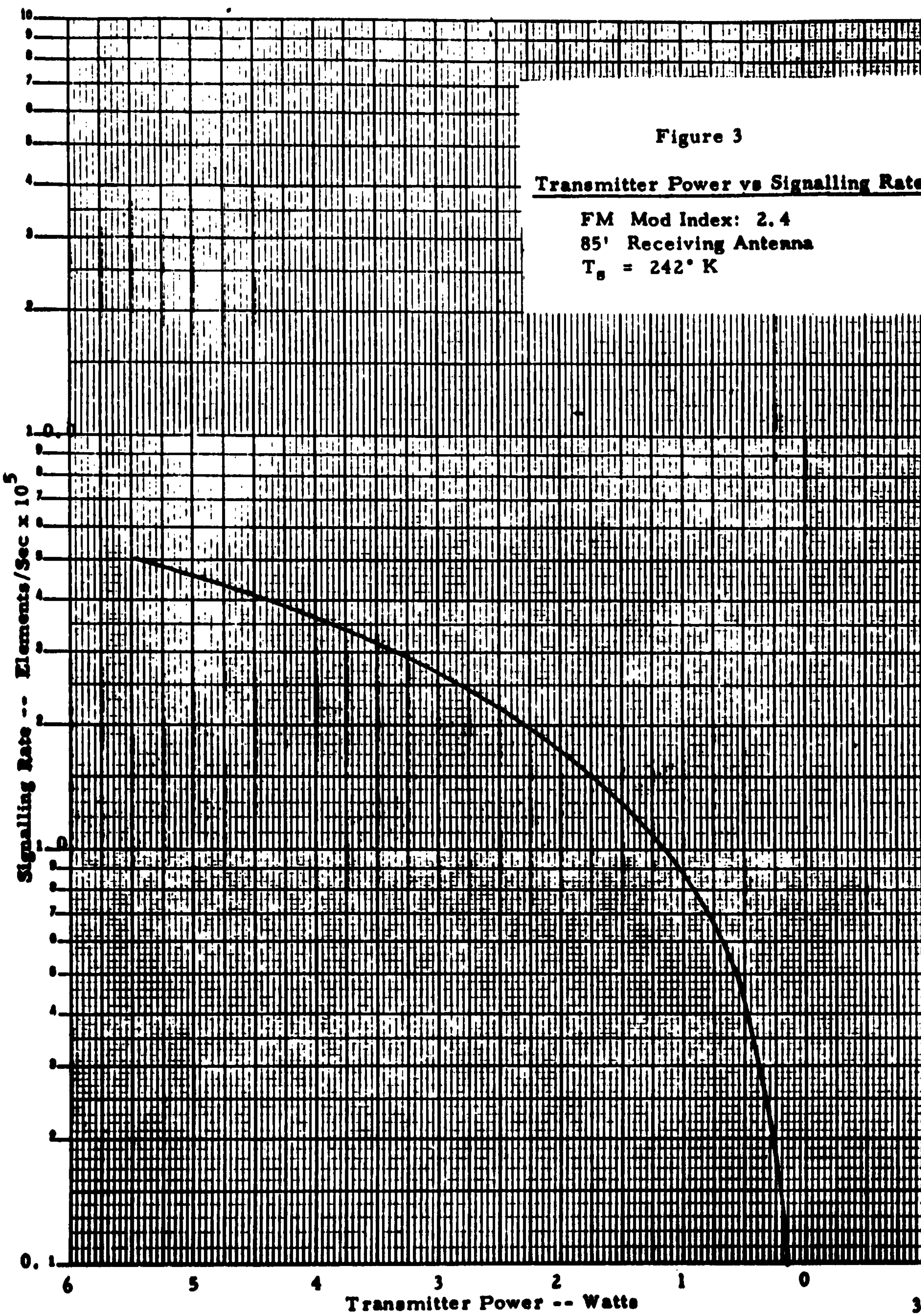


Figure 3
Transmitter Power vs Signalling Rate
FM Mod Index: 2.4
85' Receiving Antenna
T_s = 242° K
Signalling Rate -- Elements/Sec x 10^5
10
1.0
0.1
6
5
4
3
2
1
0
Transmitter Power -- Watts

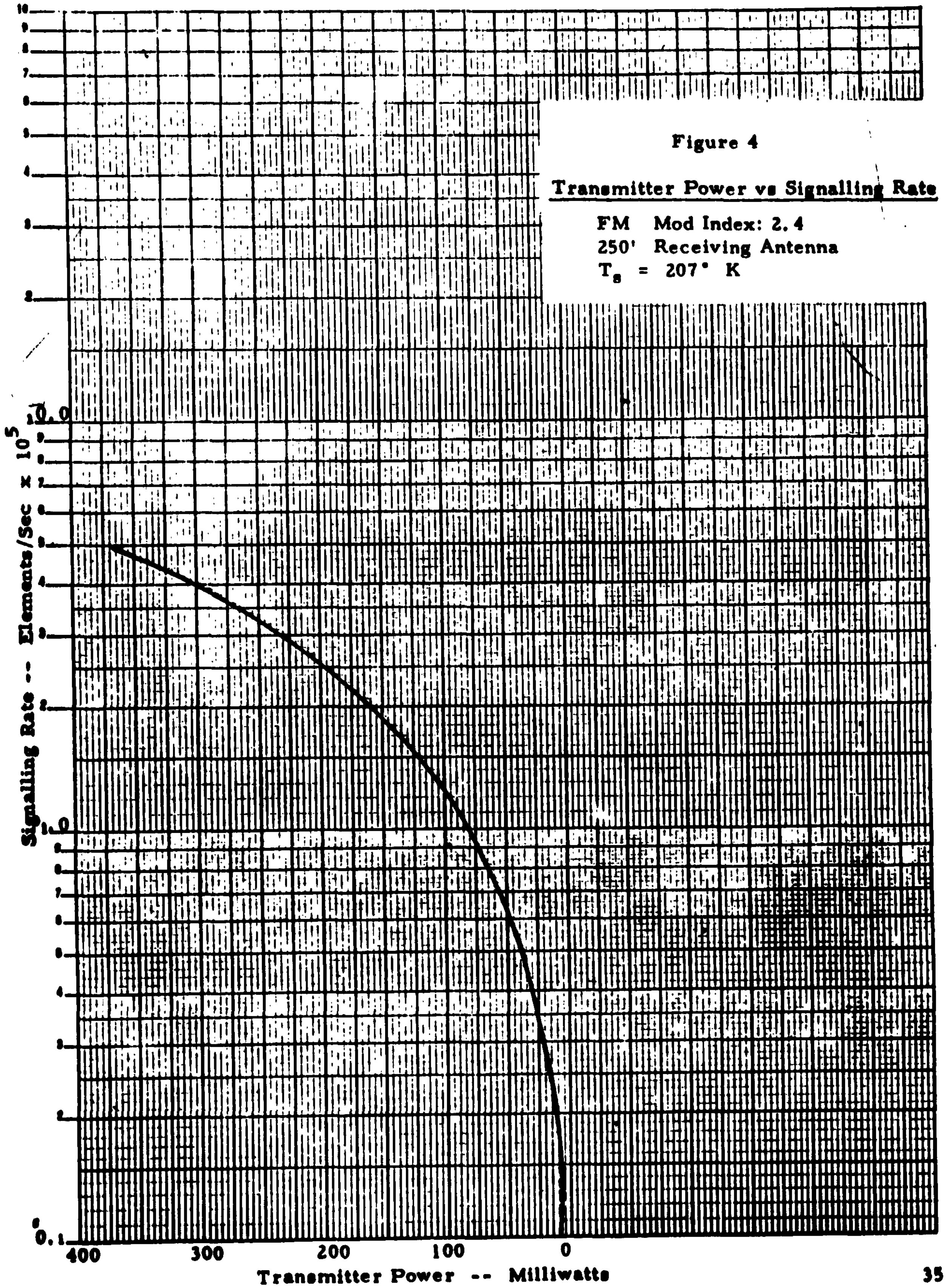


Figure 4
Transmitter Power vs Signalling Rate
FM Mod Index: 2.4
250' Receiving Antenna
T_s = 207° K
Signalling Rate -- Elements/Sec x 10^5
10.0
1.0
0.1
400
300
200
100
0
Transmitter Power -- Milliwatts

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3. JPL Research Summary 36-7, Vol. 1, Feb. 15, 1961. Page 78.

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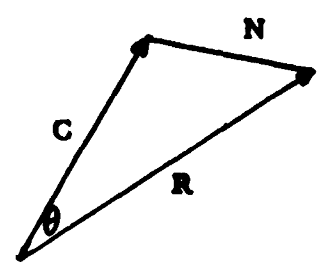
APPENDIX I

S/N Ratio in an FM System

Assume (1) a carrier with a peak amplitude C (unmodulated), and angular velocity w; and (2) a noise component of peak amplitude N with a momentary angular velocity p. This leads to the summation of the two rotating vectors Ce^{-jwt} and Ne^{jpt}. Now the resultant phase angle can be taken and wt subtracted from it to get the noise contribution to phase angle, or θ can be evaluated directly from the diagram.



The first case becomes:

$$\Psi = \tan^{-1} \frac{\sin wt + x \sin pt}{\cos wt + x \cos pt} \qquad (1)$$

where $x = \frac{N}{C}$

of $$\theta = \tan^{-1} \frac{x \sin (p - w) t}{1 + x \cos (p - w) t} \qquad (2)$$

In either case the expression

$$fd = \frac{1}{2\pi} \frac{d}{dt} (\theta \text{ or } \Psi) \qquad (3)$$

can be used for the contribution of the noise:

$$fd = \left[\frac{1}{2\pi} (p - w) + (p - w) \frac{\frac{N}{C}^2 + \frac{N}{C} \cos (p - w)^t}{1 + \frac{N}{C}^2 + 2\frac{N}{C} \cos (p - w)^t}\right] \quad (4)$$

The first term in the square bracket represents a direct current which can be ignored. The second term may be calculated to give:

$$\begin{aligned}
\frac{1}{2\pi} \Bigg\{ & (p - w) \left[(\tfrac{N}{C})^2 - (\tfrac{N}{C})^4 + (\tfrac{N}{C})^6 - (\tfrac{N}{C})^8 - - - - \right. \\
& + (p - w) \cos (p - w)^t \left[(\tfrac{N}{C}) - 3(\tfrac{N}{C})^3 + 5(\tfrac{N}{C})^5 - 7(\tfrac{N}{C})^7 + 9(\tfrac{N}{C})^9 - - - \right. \\
& - (p - w) \cos^2 (p - w)^t \left[2(\tfrac{N}{C})^2 - 8(\tfrac{N}{C})^4 + 18(\tfrac{N}{C})^6 - 32(\tfrac{N}{C})^8 + 50(\tfrac{N}{C})^{10} - - - \right. \\
& + (p-w) \cos^3 (p-w)^t \left[4(\tfrac{N}{C})^3 - 20(\tfrac{N}{C})^5 + 56(\tfrac{N}{C})^7 - 120(\tfrac{N}{C})^9 + 220(\tfrac{N}{C})^{11} - - - \right. \\
& - (p-w) \cos^4 (p-w)^t \left[8(\tfrac{N}{C})^4 - 48(\tfrac{N}{C})^6 + 104(\tfrac{N}{C})^8 - 224(\tfrac{N}{C})^{10} + - - - - - \right. \\
& + (p-w) \cos^5 (p-w)^t \left[16(\tfrac{N}{C})^5 - 64(\tfrac{N}{C})^7 + 168(\tfrac{N}{C})^9 - - - \right. \\
& - (p-w) \cos^6 (p-w)^t \left[32(\tfrac{N}{C})^6 - 160(\tfrac{N}{C})^8 + 328(\tfrac{N}{C})^{10} - - - \right. \\
& - (p-w) \cos^7 (p-w)^t \left[64(\tfrac{N}{C})^7 - - - - \right. \Bigg\}
\end{aligned} \quad (5)$$

The first terms give added direct current due to the difference frequency. The main term, if N/C is small, is

$$\frac{N}{C}(p-w)\cos(p-w)t$$

which shows that a noise component gives an output proportional to N/C and proportional to its spacing from the carrier frequency.

Now consider an i-f amplifier in which a signal-to-thermal noise power ratio of C/N has been calculated. Then by taking an interfering signal at a frequency f from the carrier (considering it one noise component), the power will be $n = N/Bif$. The ratio of amplitude

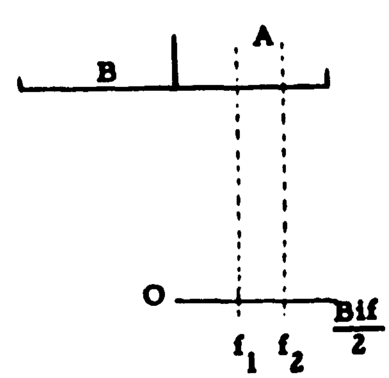
$$= \sqrt{\frac{n}{C}}$$

From the analysis above, the interfering carrier will produce an output of

$$K\sqrt{\frac{n}{C}}\,f$$

where K is the demodulator transfer constant. Now if the frequency of the interfering carrier is varied throughout the i-f band, we find that an output is produced between f1 and f2 when the carrier is in the A region and also when it is in the B region. The power per cycle in the f1 to f2 range from the multiplicity of carriers comprising the noise in the A region is

$$= K^2 \frac{n}{C} f^2$$



The noise power appearing in band f1 to f2 is:

$$2 K^2 \frac{n}{C} \int_{f_1}^{f_2} f^2 \, df = 2 K^2 \frac{n}{C} \frac{1}{3} (f_2{}^3 - f_1{}^3) \qquad (8)$$

where 2 takes care of both A and B regions.

The signal in the output will be $K \Delta F$ in amplitude, or $K^2 \Delta F^2$ in power, so the output signal to noise ratio will be:

$$\frac{K^2 (\Delta F)^2 \, 3}{2 K^2 (f_2{}^3 - f_1{}^3)} \quad \frac{C}{n} = \frac{C}{N} \quad \frac{Bif}{2(f_2 - f_1)} \quad \frac{3 (\Delta F)^2}{(f_2{}^2 + f_1 f_2 + f^2)} \qquad (9)$$

Summary

$$S/N \text{ (rms)} = \frac{C}{N} \quad \frac{Bif}{2(f_2 - f_1)} \quad \frac{3 (\Delta F^2)}{(f_2{}^2 + f_1 f_2 + f^2)} \qquad (10)$$

Since in a wide band modulation system $f_2 >> f_1$,

$$S/N \text{ (rms)} = \frac{C}{N} \quad \frac{Bif}{2(f_2 - f_1)} \quad \frac{3 (\Delta F^2)}{(f_2{}^2)} \qquad (11)$$

If we let $f_2 - f_1 = Bv$ the video bandwidth, and $f_2 = f_t$ the highest video frequency,

$$S/N \text{ (rms)} = \frac{C}{N} \quad \frac{Bif}{2(B_v)} \quad 3 \left(\frac{\Delta F}{f_t} \right)^2 \qquad (12)$$

To convert to db we multiply each side of Equation (12) by $10 \log_{10}$ to obtain:

$$S/N\ db = 10 \log_{10} \frac{C}{N} + 10 \log_{10} \frac{Bif}{2\ B_v} + 5\ db + 20 \log_{10} \left(\frac{\Delta F}{f_t}\right) \quad (13)$$

Equation (13) assumes perfect limiting, if we assume a limiter efficiency of 50%. Equation (13) then becomes

$$S/N = 10 \log_{10} \frac{C}{N} + 10 \log_{10} \frac{Bif}{2B_v} + 20 \log_{10} \left(\frac{\Delta F}{f_t}\right) + 5\text{-}3\ db \quad (14)$$